BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

B70 12044

SUBJECT: Stability of a Spinning Spacecraft with Extended Flexible Booms - Case 620

DATE: December 18, 1970

FROM: L. E. Voelker

ABSTRACT

An artificial gravity experiment has been proposed for Skylab B. Two ballast masses on deployable booms have been suggested as a means of modifying Skylab A's inertia properties to locate the stable spin axis perpendicular to the planar solar arrays.

A previous stability analysis of a rotating spacecraft with flexible solar arrays indicated that not only must rigid body stability be assured but also that the spacecraft cannot rotate faster than a certain critical rate. This analysis is extended to the vehicle with flexible booms. This case differs from the case with solar arrays because of the effect of the tension in the boom on its stiffness. The minimum first natural frequency of the boom can be found by solving a transcendental relationship. For the ballasted Skylab B, the first boom natural frequency must be greater than 61% of the spin rate for dynamic stability.

(NASA-CR-116354) STABILITY OF A SPINNING SPACECRAFT WITH EXTENDED FLEXIBLE BOOMS (Bellcomm, Inc.) 9 p

N79-72410

00/18 Unclas 11997

No. CR-116354
(NASA CR OR TMX OR AD NUMBER) (CATEGORY)
[redacted]

BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

B70 12044

SUBJECT: Stability of a Spinning Spacecraft with Extended Flexible Booms - Case 620

DATE: December 18, 1970

FROM: L. E. Voelker

MEMORANDUM FOR FILE

Artificial gravity would be achieved during an experiment with Skylab B by rotating the spacecraft about its center of mass. Passive stability of a rigid rotating spacecraft is assured only if the spacecraft is rotating constantly about its axis of maximum moment of inertia. The Skylab solar arrays should be normal to the sun line for maximum efficiency. Therefore, the axis of maximum moment of inertia of the Skylab should coincide with the spacecraft Z-axis, which is the solar array normal.

Shown in Figure 1 is a proposed configuration of Skylab [1] which utilizes ballast masses on offset deployable booms to align the principal axes to within 4° of the spacecraft axes. Since deployable booms cannot be considered rigid, the passive stability of a rotating spacecraft with such booms is of concern. The stability conditions for an idealized model of a spacecraft with symmetric solar arrays in asymmetric flexure have been dervied [2]. The conditions require that the angular velocity vector must coincide with the axis of maximum moment of inertia of the spacecraft and that the rate of rotation must be less than a certain critical rotation rate.

Figure 2 shows an idealized model of the Skylab with offset booms. The xyz axes are the principal axes of the vehicle with corresponding moments of inertia I_x, I_y, I_z. It is assumed that the lowest natural frequency of the booms will govern the stability of the spacecraft so the Skylab with the booms removed is treated as a rigid body. Each boom is of length b, and is considered massless, the boom mass being incorporated into the ballast mass, m, and into the rigid body. The masses are restricted to undergo antisymmetric displacements in the z-direction only. The ballast masses and booms are modeled as mass-spring-damper systems mounted in rigid massless frames. The spacecraft is rotating about its z-axis at a nominal rotation rate r_o.

The linearized equations of motion for this model are identical in form to those of Reference 2. However, the stiffness K of the solar arrays of Reference 2 is constant, whereas the tension in the booms will increase their effective stiffness. For an axial load T, the effective stiffness of a boom of length b may be written as [3]

$$K = \frac{T}{b} \left(1 - \frac{\tanh k}{k}\right)^{-1}$$

where $k^2 = Tb^2/EI$, EI being the product of the Young's modulus and the area moment of inertia of the boom. The axial tension linearizes to $T = mbr_o^2$ so the square of the frequency of the boom becomes

$$\omega_p^2 = \frac{K}{m} = r_o^2 \quad \left(1 - \frac{\tanh k}{k}\right)^{-1}$$

and $k^2 = 3r_o^2/\omega_n^2$, where $\omega_n^2 = 3EI/mb^2$ is the natural frequency of a cantilevered beam with mass m at its end, which can be easily obtained from the Strength of Materials load versus deflection relation [4].

It is probable that the rotation rate, r_o, will be set by considerations such as desired g-levels, physiological factors and spacecraft configuration. Therefore, it will be required to design the ballast booms based on a given r_o so that the configuration is dynamically stable. To this end, a minimum or limiting first natural frequency of the boom will be derived. The limiting condition on boom natural frequency is, from Reference 2

$$(I_z - I_y)\,\omega_p^2 > I_b r_o^2$$

where $I_b = 2mb^2$. The limiting natural frequency, ω_ℓ, is just

$$\omega_\ell^2 = I_b r_o^2/(I_z - I_y)$$

In Reference 2 the stiffness of the solar array is constant and $\omega_p = \omega_n$. In this problem, as we have seen, the tension in the booms increases their stiffness. Substitution of the square of spin rate, r_o^2, yields the limiting boom natural frequency:

$$\omega_\ell^2 = I_b\ \omega_p^2\ \left(1 - \frac{\tanh k}{k}\right) / (I_z - I_y).$$

By letting $\omega_p = \omega_\ell$ in the above equation, the following transcendental relationship results:

$$\frac{\tanh k}{k} = 1 + (I_y - I_z)/I_b$$

$$= (I_y + I_b - I_z)/I_b.$$

where $$k^2 = 3r_\circ^2/\omega_\ell^2$$

A solution of this transcendental equation for k will give the ratio of limiting boom natural frequency to rotation rate through the above expression for k. It is emphasized that ω_ℓ is the limiting boom natural frequency without axial tension present. Figure 3 shows a plot of (tanh k)/k vs k which can be used to find approximate values of $r_\circ/\omega_\ell$. For (tanh k)/k < 0.4, k > 2.5, and tanh k can be replaced by 1. The limiting boom first natural frequency can then be found directly from

$$\omega_\ell = \sqrt{3} \quad \frac{(I_y + I_b - I_z)}{I_b} r_\circ, \ \tanh k/k < 0.4.$$

For the vehicle shown in Figure 1,

$$(I_y + I_b - I_z)/I_b = 0.35$$

and $\omega_\ell = 0.61\ r_\circ$. The stability analysis therefore indicates that the ballast booms used to modify the Skylab inertia properties for artificial gravity must have a first natural frequency, greater than 61% of the spin rate for dynamic stability.

Acknowledgement

The suggestions of R. J. Ravera on the content of this memo are appreciated.

L. E. Voelker

1022-LEV-tla L. E. Voelker

Attachments
References
Figures 1-3

BELLCOMM, INC.

REFERENCES

1. Voelker, L. E., "Spin-up of Skylab B for Artificial Gravity", Bellcomm Technical Memorandum 70-1022-14, Sept. 30, 1970.

2. Fearnsides, J. J., Ravera, R. J., and Voelker, L. E., "Stability of a Spinning Spacecraft with Flexible Solar Arrays", Bellcomm Memorandum for File, November 4, 1970.

3. Ziegler, H., "Principles of Structural Stability", Blaisdell Publishing Co., 1968, pp. 87-89.

4. Timoshenko, S. and Young, D. H., "Elements of Strength of Materials", D. Von Nostrand Co., Princeton, N. J., 1962, p. 212.

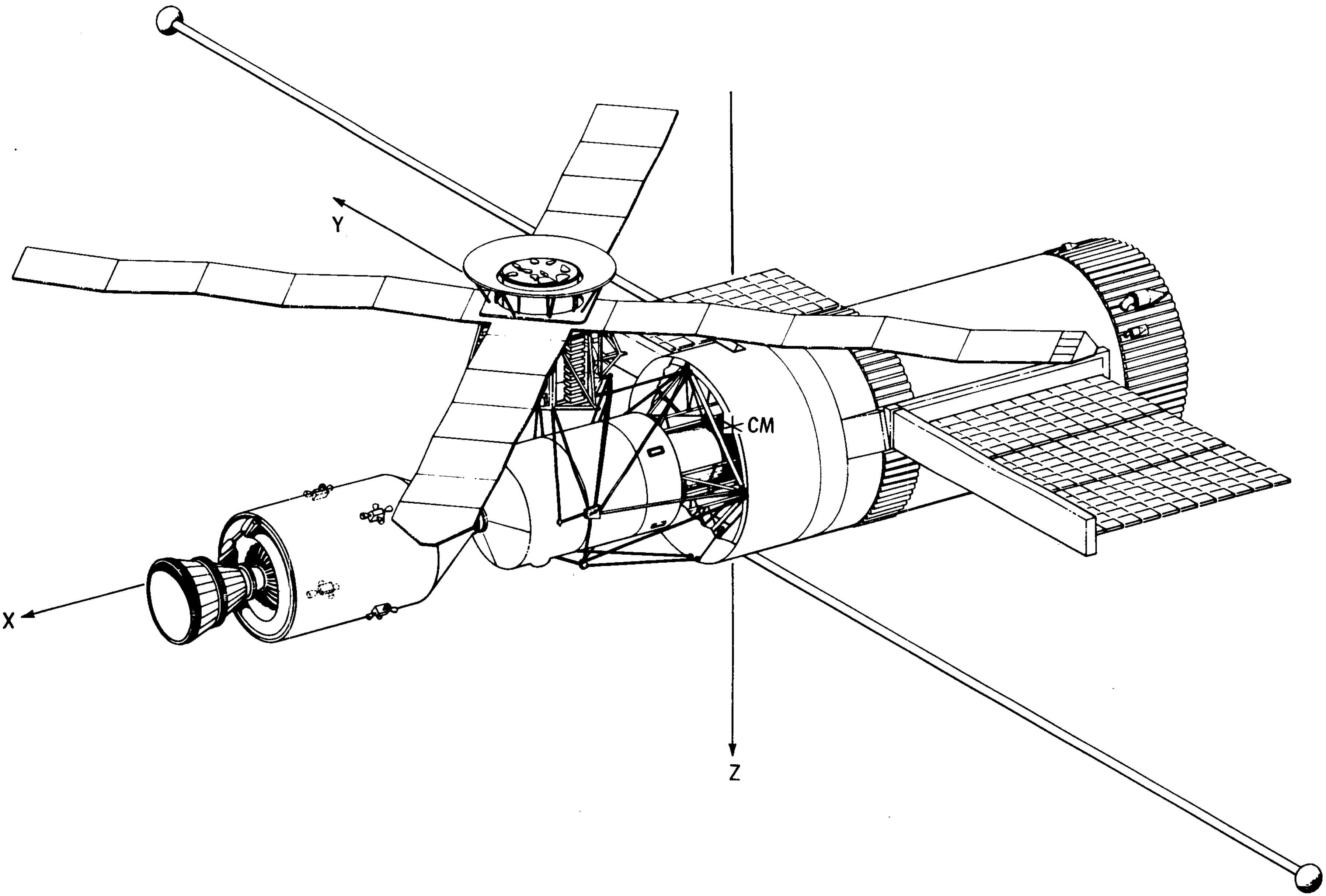

FIGURE 1 - SKYLAB BALANCED FOR ARTIFICIAL GRAVITY EXPERIMENT

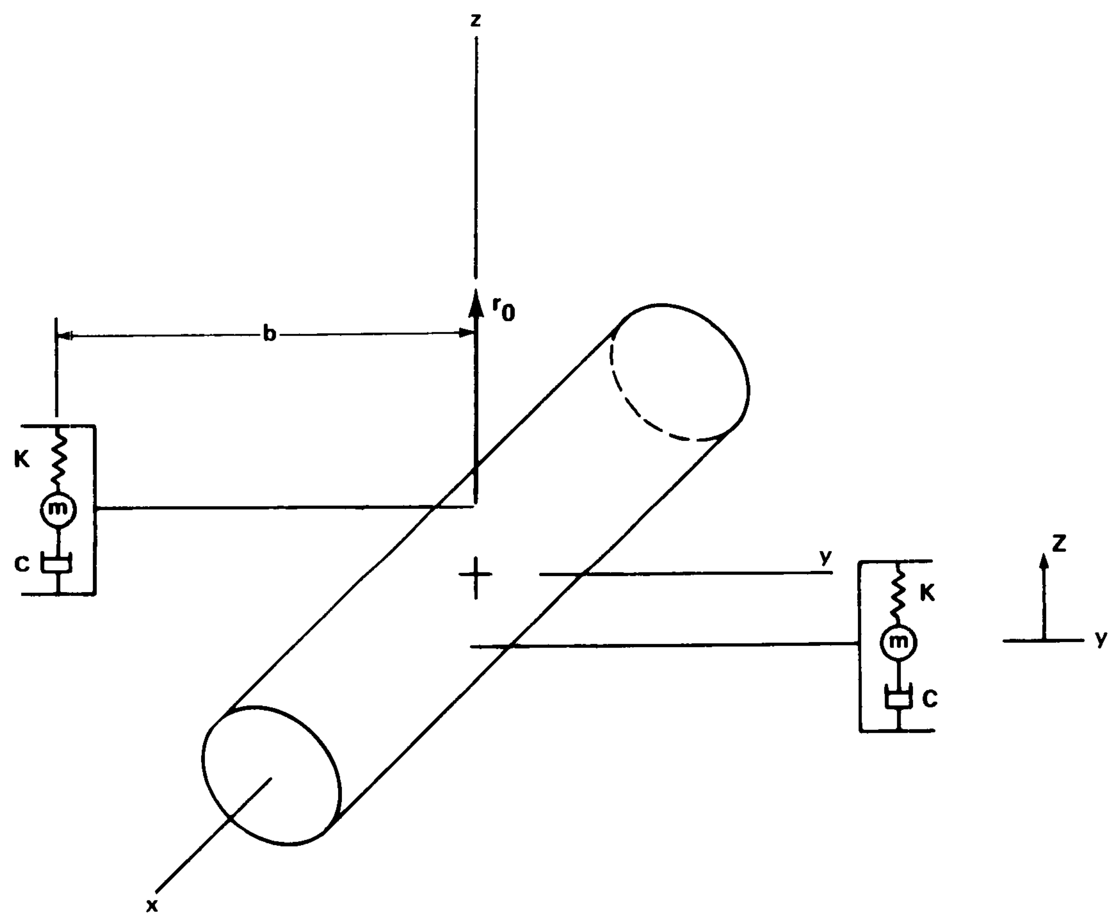

FIGURE 2 - MODEL OF SKYLAB WITH SYMMETRIC FLEXIBLE BOOMS

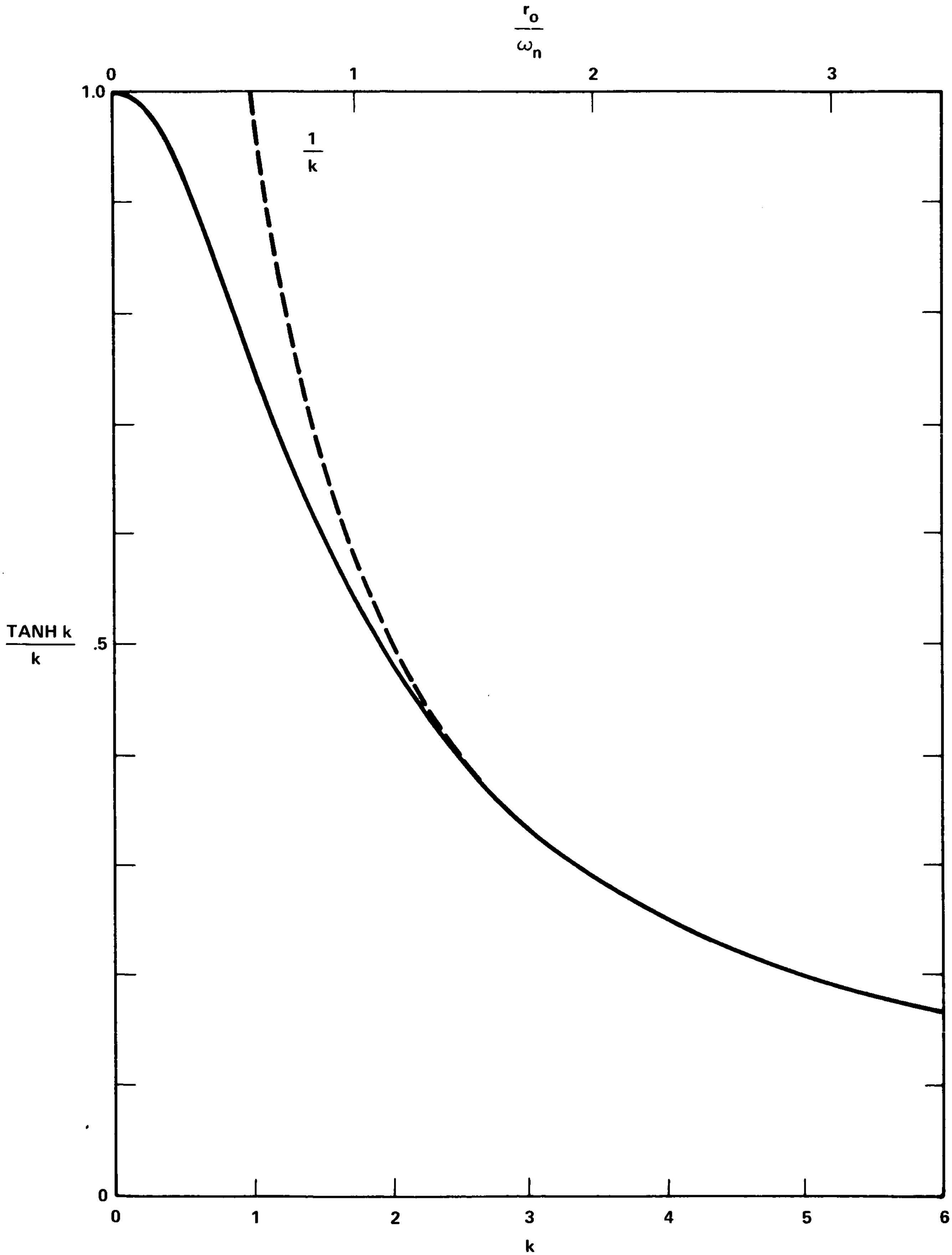

FIGURE 3 - TANH k/k VS k CURVE FOR DETERMINING THE MINIMUM BOOM FIRST NATURAL FREQUENCY, ω_ℓ

BELLCOMM, INC.

Subject: Stability of a Spinning Spacecraft with Extended Flexible Booms
Case 620

From: L. E. Voelker

Distribution List

NASA Headquarters

H. Cohen/MLR
J. H. Disher/MLD
W. B. Evans/MLO
J. P. Field, Jr./MLP
T. E. Hanes/MLA
A. S. Lyman/MR
M. Savage/MLT
W. C. Schneider/ML

MSC

K. J. Cox/EG-23
K. L. Lindsay/EG-23
C. F. Lively, Jr./EG-23
A. J. Louviere/EW-6

NAR/Downey

J. A. Jansen/BB-48

Bellcomm

A. P. Boysen
J. P. Downs
D. R. Hagner
W. G. Heffron
D. P. Ling
J. Z. Menard
I. M. Ross
P. F. Sennewald
J. W. Timko
R. L. Wagner
M. P. Wilson
Departments 2031, 2034 Supervision
Division 101 Supervision
Department 1024 File
Division 102
Central File
Library